International Journal of Thermophysics, Vol. 11, No. 1, 1990

The Permeability of Alumina over an Extended Temperature Range[1]

W. W. Pulkrabek,[2] R. M. Wabrek,[3] and W. E. Ibele[4]

A semiempirical mathematical model has been developed to predict the permeability of porous alumina over a wide temperature range at atmospheric pressure. Although the thermal expansion of the alumina is of the order of 1% over the range of temperatures considered, the apparent permeability of the sample to gas flow varies by over 400%. This behavior is due to the dependence of the mean free path of a gas on temperature and to the corresponding slip conditions that occur in the pores of the solid. The model developed correlates the "apparent permeability" data with temperature, true permeability, gas viscosity, and gas molecular weight, variables suggested by kinetic theory on mean free path and slip. Apparent permeability was found to be a very strong function of temperature. It exhibited both a direct thermal dependence and an indirect dependence, manifest through thermally driven variations in the gas viscosity. A mathematical model from the literature was used for gas viscosity. The inverse correlation with gas molecular weight, suggested by kinetic theory, is demonstrated. The model covers the temperature range between 250 and 1600 K. Small molelcule gases including air, nitrogen, argon, and helium were used in the development.

KEY WORDS: high-temperature gas flow; permeability; porous ceramics; slip flow.

[1] Paper presented at the Tenth Symposium on Thermophysical Properties, June 20–23, 1988, Gaithersburg, Maryland, U.S.A.

[2] Mechanical Engineering Department, University of Wisconsin—Platteville, Platteville, Wisconsin 53818, U.S.A.

[3] General Engineering Division, University of Wisconsin—Platteville, Platteville, Wisconsin 53818, U.S.A.

[4] Heat Transfer Laboratory, Mechanical Engineering Department, University of Minnesota, Minneapolis, Minnesota 55455, U.S.A.

0195-928X/90/0100-0251$06.00/0

1. INTRODUCTION

In recent years, the flow of compressible fluids through high-temperature porous media has become a more important phenomenon in engineering work. Applications include flow in catalytic converters, mass transfer cooling of turbine blades, and gas injection into the arc region of high temperature plasmas. Until recently, very little information on the temperature dependence of the permeability of a porous material has appeared in the literature. The only data available [1–6] describe the temperature range between 0 and 60°C.

Recent work [7] involving compressible flow through porous consolidated aluminum oxide ceramic (alumina) has shown that the apparent permeability of a material is highly dependent upon temperature. In response to changes in temperature between 250 and 1600 K, permeability has been observed to change by over 400%. This behavior is not caused so much by a variation in the actual permeability but, rather, by a variation in the apparent permeability caused by "slip flow" in the small pores of the material. This slip occurs when the mean free path of the gas is of the same order of magnitude as the pore diameters.

Slip flow depends on the mean free path of the gas molecules, and this free path is strongly dependent upon temperature. Thus, even though the actual physical characteristics of the material, including the permeability, change very little with temperature, the apparent permeability can experience very large changes. Kinetic theory indicates that the mean free path of a gas is dependent on temperature and viscosity, with viscosity itself a strong function of temperature.

The goal of this work was to develop a mathematical model of the apparent permeability of the porous medium as a function of temperature. The model was established by using data on three different gases, with both monatomic and diatomic gases represented. Molecular weights of the test gases covered an order-of-magnitude range. The model was developed with the goal of predicting the apparent permeability for flow of other gases in the same molecular weight range.

2. FLOW THROUGH POROUS MEDIA

Modern one-dimensional flow equations for fluids passing through porous media originated with work done by Darcy in 1856 [8]. Darcy's equation was developed for water flowing through beds of sand and is meant to be applied at low flow rates. In differential form, it can be written as

$$-k\frac{dP}{ds}=\mu V \tag{1}$$

where μ is the dynamic viscosity of the fluid, V is the average velocity of the flow, and dP is the differential pressure drop that occurs over the differential length, ds. This equation provides a definition of the parameter, k, the permeability of the porous medium. The permeability is reasoned to be a property of the solid porous medium, independent of the flowing fluid. Permeability would be expected to depend on suich properties as porosity, pore size, pore shape distribution, and surface area, among others. Through the mechanism of thermal expansion, permeability would also depend on temperature.

Although history confirms that Darcy's equation very accurately predicts one-dimensional flow at low flow rates for most combinations of incompressible fluids and porous media, the equation falters when applied to compressible fluids. Permeability values calculated on the basis of gas flow through a specific medium were consistently higher than values based on liquid flow. In addition, permeability calculated on the basis of the flow of one gas would differ from that based on another gas flowing throught the same medium. Further, for a single gas, calculated values of porous-medium permeability varied with gas temperature. All these phenomena may be attributed to the slip flow that occurs in the very small pores.

When the apparent permeability of a porous aluminum oxide ceramic was measured over a large temperature range [7], there were large differences between gases and an obvious temperature dependence for each individual gas (see Fig. 1). Although the thermal expansion of the medium was only 1.03%, the ostensible permeability varied by over 400%.

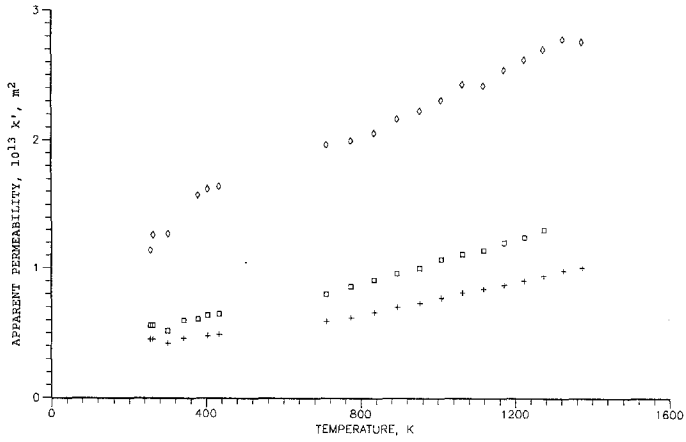

Fig. 1. Experimental values of apparent permeability of alumina for three gases. (□) Argon; (+) N_2; (◇) helium [7].

3. MATHEMATICAL MODEL

The objective of this work was to develop a semiempirical mathematical model which would describe the experimental behavior of certain gases at 1-atm pressure, between the temperatures of 250 and 1600 K. A successful model could then be used to predict the apparent permeability for other gases.

Slip in gas flow through a porous medium has the same effect as it does in capillary flow. The mass flow rate will exceed that predicted by Darcy's equation for the measured pressure drop. If the porous-medium flow is modeled in a similar manner to capillary flow, then the permeability, k in Eq. (1), should be replaced by an apparent permeability k' defined by

$$k' = k(1 + C_1 \lambda) \tag{2}$$

where λ is the mean free path of the gas molecules, and C_1 is a constant or possibly a temperature-dependent parameter.

From basic kinetic theory,

$$\lambda = C_2 \frac{\mu}{P} \sqrt{T/M} \tag{3}$$

where T is the temperature, P the pressure, and M the molecular weight of the gas. C_2 is a constant. Combining Eq. (2) with Eq. (3) gives

$$k' = k(1 + C_3 \mu \sqrt{T/M}) \tag{4}$$

Since pressures were held approximately constant near 1 atm, the pressure term has been eliminated in Eq. (4) by including it within the parameter C_3. Actual measured pressures ranged from 0.95 to 1.12 atm.

Although Eq. (4) does describe the general behavior of k' as a function of each variable, it would not accurately describe the behavior of individual gases over a wide temperature range. This led to attempts to modify the basic equation so that it would more accurately fit the experimental data for specific gases. Equation (5) represents the most successful modification to this basic kinetic theory model.

$$k' = k(1 + C\mu^A M^B) \tag{5}$$

In Eq. (5) the temperature term was combined with the parameter C_3 to form C, a dimensionless temperature-dependent variable. In initial

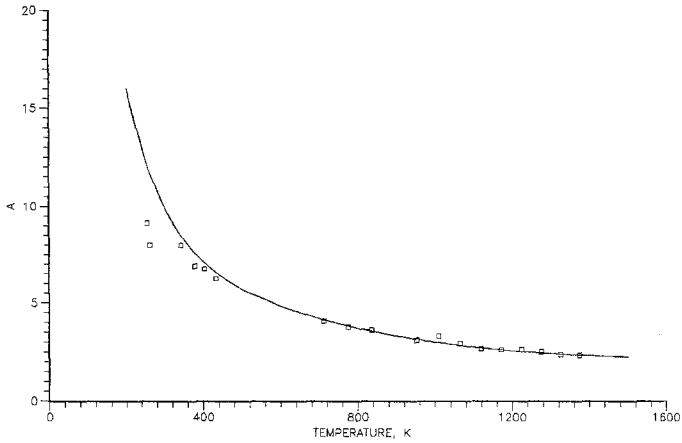

Fig. 2. Exponent A from Eq. (5) as a function of temperature. (□) Experimental.

attempts, the dimensionless exponents A and B were constrained to be constants. This proved unsuccessful, and it was necessary to allow these parameters to vary with temperature.

Using experimental data for helium, argon, and nitrogen, the A, B, and C parameters were determinied at various temperatures. Viscosity values in $10^{-7}\,\mathrm{N \cdot s \cdot m^{-2}}$ were obtained from Ref. 9, which gives a model that correlates much of the known data. This viscosity model is also a strong function of temperature.

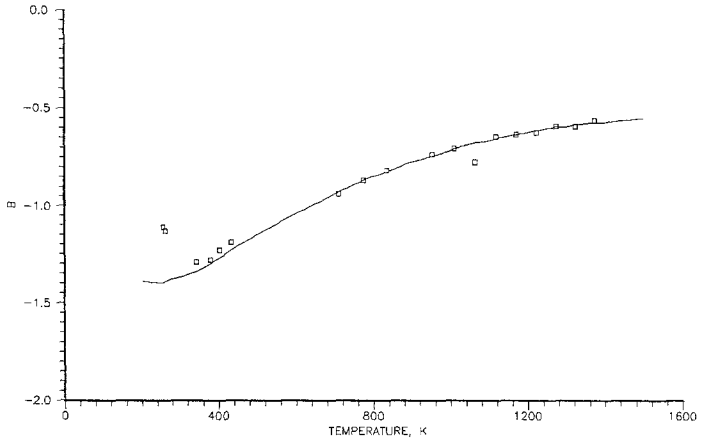

Fig. 3. Exponent B from Eq. (5) as a function of temperature. (□) Experimental.

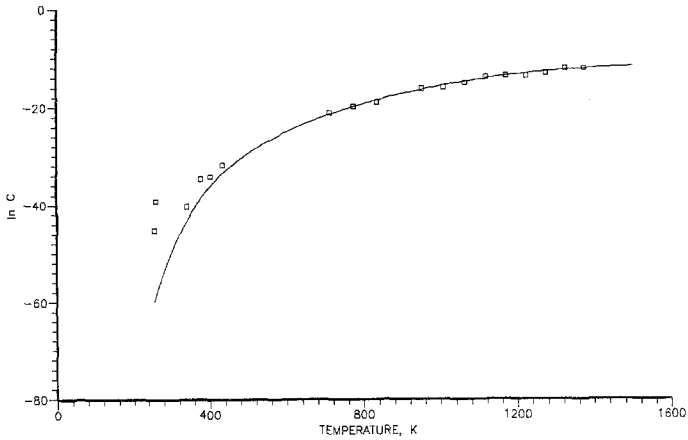

Fig. 4. Parameter C from Eq. (5) as a funuction of temperature. (□) Experimental.

4. RESULTS

Figures 2 through 4 show the temperature dependence of the parameters A, B, and C. Curves of the form

$$A = N_0 + N_1 \exp[0.01\ (1600 - T) - 1] + N_2 \exp[0.001\ (1600 - T) - 1] + N_3 \exp[0.0001\ (1600 - T) - 1] \quad (6)$$

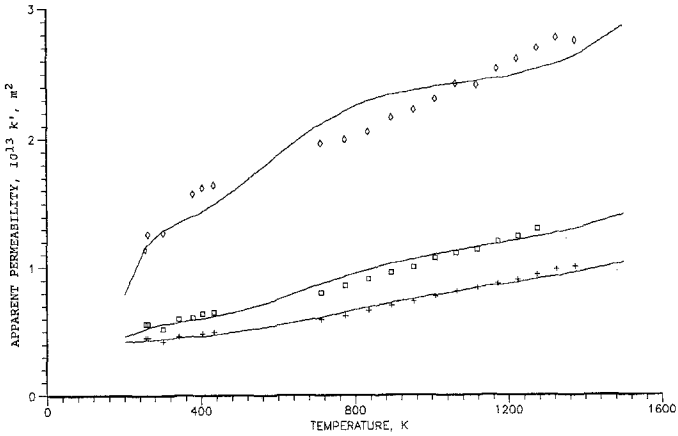

Fig. 5. Apparent permeability, k', of alumina predicted by Eq. (5), for gases used in model development. (□) Argon; (+) N_2; (◇) helium [7].

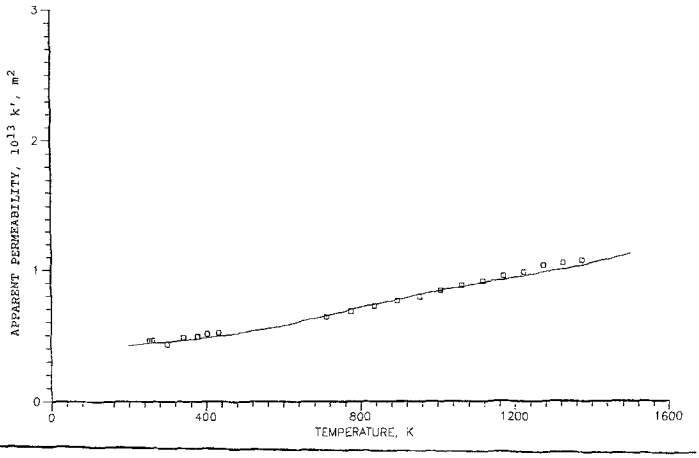

Fig. 6. Apparent permeability of alumina for flow of air. (□) Experimental [7].

were fit to A, B, and ln C, with temperature, T, in K. The values of the dimensionless N_i terms are given in Table I. Using these values in Eq. (5) provides predictions for the permeability of the reference gases, helium, argon, and nitrogen; see Fig. 5.

Equation (5) predicts the apparent permeability for nitrogen and argon quite well and gives acceptable results for helium. The accuracy of the helium data fit could be significantly improved by adding more terms to Eq. (6). This amounts to a trade-off—improved accuracy at the expense of a more complicated model. Equation (6) represents the resulting compromise.

Since air data were not used in the development of the empirical model, the model was applied to the flow of air through aluminum oxide as a test. The results were enouraging (Fig. 6). The model predicts the permeability for air flow to within an average of 3.5% of the measured data over the entire temperature range.

Table I. Values of Dimensionless Constants for Eq. (6)

Parameter	N_0	N_1	N_2	N_3
A	2.233	0.6817×10^{-5}	3.558	−35.09
B	−0.5534	0.2246×10^{-6}	−0.6658	6.191
ln C	−11.81	-0.3166×10^{-4}	−18.10	176.2

5. CONCLUSIONS

Results of this work indicate that for gas flow through a porous aluminum oxide solid at 1-atm pressure, Eq. (1) will accurately predict the flow rates if the permeability k is replaced with the apparent permeability k', as given in Eq. (5). The coefficients for Eq. (5) are obtained from Eq. (6) and Table I.

REFERENCES

1. J. C. Calhoun, Jr., and S. T. Yuster, *A.P.I. Drill. Prod. Prac.* 335 (1946).
2. H. Krutter and R. J. Day, *Oil Week.* **104**:24 (1941).
3. M. B. Biles and J. A. Putnam, NACA Tech. Note 2783 (1952).
4. P. C. Carman, *Proc. R. Soc. Lon.* **A203**:55 (1950).
5. D. B. Greenberg and E. Weger, *Chem. Eng. Sci.* **12**:8 (1960).
6. T. W. Johnson and D. B. Taliaferro, Bur. Mines Tech. Paper No. 592 (1938).
7. W. W. Pulkrabek and W. E. Ibele, *Int. J. Heat Mass Trans.* **30**:1103 (1987).
8. M. Muskat, *The Flow of Homogeneous Fluids Through Porous Media* (J. W. Edwards, Ann Arbor, Mich. 1946), pp. 55–120.
9. G. C. Maitland and E. B. Smith, *J. Chem. Eng. Data* **17**:150 (1972).